Turbulence in the Interstellar Medium

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1 Introduction

It is known today that turbulence is prevalent in the interstellar medium (ISM). Turbulence affects the structure and motions of nearly all temperature and density regimes in the interstellar gas. A similar theory to what astronomers believe today was developed about turbulence in interstellar matter in 1951. Von Weizsacker developed a theory stating that cloudy objects with a hierarchy of structures form in noninteracting shock waves by supersonic turbulence that is stirred on the largest scale by differential galactic rotation, where parts of the rotating galaxy move with different angular velocity, and dissipated on small scales by atomic viscosity. Also in 1951, von Hoerner observed rms velocity differences in emission lines of the Orion nebula that increased with projected separation as a power-law reminiscent of a turbulent gas with a Kolmogorov energy cascade. Wilson et al. got a similar result in 1959, but proposed that their results suggested compressible turbulence. Motions that correlated with a Kolmogorov structure function were observed by Kaplan in 1959 in optical absorption lines. However, the idea of turbulence being important in the ISM was suspect to many scientists when it was first proposed.

Interstellar absorption and emission lines looked too smooth to come from turbulent structures. Spitzer thought that the shapes of objects in the ISM appeared wrong for turbulence. ISM models without turbulence could model most of the interesting physical properties well enough. They allowed easily interpreted results that were not possible if turbulence was introduced. Various observations did not give any sort of compelling reason to change the cloud-intercloud model in favor of turbulence. The skeptics had a different model of turbulence than the one used today. However, turbulence on smaller scales was more widely accepted. Observations of interstellar scintillation (twinkling) at radio wavelengths implied turbulence in ionized gas at scales below 10^9 cm. Turbulence was beginning to be more accepted when Larson (1981) found power-law correlations between molecular cloud sizes and linewidths that seemed to correlate reasonably well with Kolmogorov's scaling law, a fundamental analytical feature of turbulence. Finally, observations in 1984 started to suggest turbulence played a large role in the ISM on a large scale, where filamentary and cris-crossed cloud patterns were seen with the Infrared Astronomical Satellite. Additionally, the power spectrum for widespread HI emission was comparable to the Kolmogorov power spectrum for velocity in incompressible turbulence. CO, the main indicator of molecular clouds, was seen to have velocities which were correlated over a range of scales as well. By the end of the eighties, compression from interstellar turbulence was seen as one of the main mechanisms for cloud formation.

The goal for the rest of this paper is to try to discuss turbulence at a basic level without specifically invoking astrophysical interpretations, eventually using this formalism to describe and discuss the turbulent ISM. Observations of the ISM showcasing turbulence will be discussed in detail, and their impact on future observations and theoretical work will be discussed. Finally, star formation will be discussed, including the historical models of star formation, as well as more modern models. Turbulence is often used as a "pressure" in star formation models, and the effects of this versus doing simulations with full blown supersonic turbulence will be discussed.

2 What is Turbulence?

Werner Heisenberg, one of the founders of quantum mechanics, responded to the question, "If given the opportunity, what would you ask God?", in which he responded, "When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first." Finding a complete description of turbulence is one of the major unsolved problems in physics. Turbulent flow is characterized by chaotic, stochastic property changes resulting in the excitation of an extreme range of correlated spatial and temporal scales. Turbulence can be roughly characterized as flows with a Reynolds number greater than 2000. The Reynolds number is defined as:

$$R \sim \left| \frac{inertial\ term}{viscous\ term} \right| \sim \frac{VL}{\nu} \tag{1}$$

where V is a characteristic speed of the flow, L is a characteristic length, and ν is the kinematic viscosity and the inertial term and viscous term come from the Navier-Stokes equation:

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{(\vec{v} \cdot \nabla)\vec{v}}_{inertial\ term} = -\frac{1}{\rho}\nabla p + \underbrace{\nu \nabla^2 \vec{v}}_{viscous\ term} + \vec{f}_{ext} \tag{2}$$

where $\vec{f}_{ext}$ is the force per unit mass (ex. $\vec{g}$). The nonlinear term $(\vec{v} \cdot \nabla)\vec{v}$ is essentially the source of hydrodynamic turbulence. Fully developed turbulence is characterized by rapid, irregular velocity fluctuations in three dimensions as well as by motions on many different length scales at once. Viscosity is usually only important on small scales, where it dissipates energy passed down from larger scales. For now, we will only talk about incompressible flow which gives the extra condition that:

$$\nabla \cdot \vec{v} = 0 \tag{3}$$

The ISM is indeed highly compressible, but the theoretical study of incompressible flow has been shown to be very useful for understanding much phenomena in turbulence.

2.1 Kolmogorov Energy Spectrum

As stated previously, the onset of turbulence usually is at high Reynolds numbers. By looking at the Navier-Stokes equations one might say that the viscous term is negligible to the inertial term, but this is only true on a certain scale. On small enough scales, the viscous term becomes comparable to the inertial term. One can understand this by considering a high Reynolds flow over a boundary. The no slip condition says that for **any** $\nu \neq 0$ the normal and tangential components of fluid velocity at a rigid boundary must be equal to those of the boundary itself. It is easily seen that for a stationary boundary, viscosity must somehow dissipate enough energy to bring the velocity of the fluid to zero at the boundary. The scale at which this occurs for a high Reynolds flow is

$$L_\nu \sim \frac{\nu}{V} \tag{4}$$

where L_ν stand for the viscous length scale and ν is the kinematic viscosity and V is the characteristic speed of the fluid as before. One can suppose that the forcing of the turbulence is only on the larger scales. Typically, there will be an instability in the flow and a smaller eddy will form from the larger one, and this process continues on to smaller and smaller scales. This is called the energy cascade. If one assumes a steady state and an energy input to the large scale, ϵ, then the energy dissipated from the smaller scales must be ϵ as well. This energy dissipation comes from viscosity on the smallest scales even at high Reynolds number. If the large forcing scale is much greater than the dissipation scale, then there is an intermediate range where neither the forcing or dissipation are important to the dynamics. This is called the inertial range. Now we will consider some velocity field decomposed into Fourier components:

$$v(x, y, z, t) = \sum_{k_x, k_y, k_z} \tilde{v}(k_x, k_y, k_z, t) e^{i(k_x t + k_y t + k_z t)} \tag{5}$$

(with $k = \sqrt{k_x^2 + k_y^2 + k_z^2} \sim 1/L$) and assume that it is statistically isotropic (the same in all directions) and homogeneous (the same everywhere). From this, one can guess a general from of the energy spectrum of the flow as a function of wavenumber:

$$\mathcal{E}(k) = f(\epsilon, k, k_0, k_\nu) \tag{6}$$

where ϵ is the energy flux, k is the wave number (notice no dependence on direction due to isotropy), k_0 is the forcing wavenumber, and k_ν is the wavenumber where viscosity becomes important (not surprisingly, $k_\nu \sim 1/L_\nu$). As stated previously, as energy cascades to smaller and smaller scales (the inertial range), the effects of the forcing are forgotten, but the effects of viscosity are not yet apparent, so the energy spectrum can be written now as:

$$\mathcal{E}(k) = f(\epsilon, k) \tag{7}$$

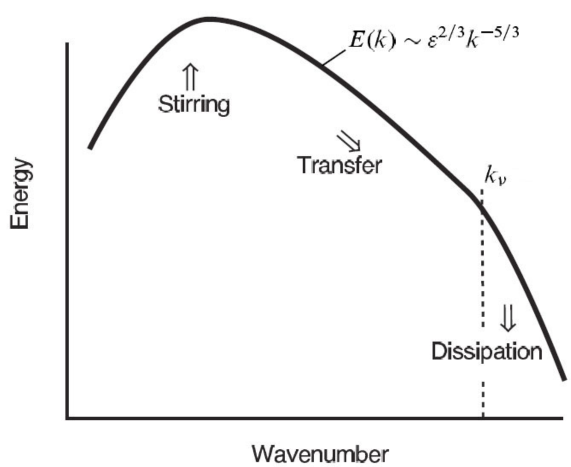
One can get an expression for the energy spectrum from dimensional analysis:

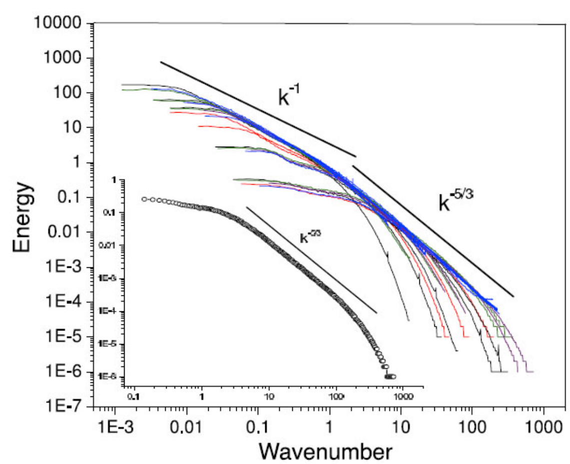
wavenumber k	$k = [1/L]$
energy per unit mass E	$E = V^2 = [L^2/T^2]$
energy flux ϵ	$\epsilon = E/T = [L^2/T^3]$
energy spectrum $\mathcal{E}(k)$	$\mathcal{E}(k) = EL = [L^3/T^2]$

where L and T denote units of length and time respectively. Given these units and that $\mathcal{E}(k) = f(\epsilon, k)$ for a isotropic, homogeneous flow, the only suitable solution is

$$\mathcal{E}(k) \propto \epsilon^{2/3} k^{-5/3} \tag{8}$$

which is Kolmogorov's famous $-5/3$ energy spectrum shown in the figure below (along with an experimentally obtained spectrum). Note that the experimentally determined proportionality constantly is approximately 1.5.





The experimental results come from the Princeton Superpipe facility and, while the smaller scale eddies closely follow Kolmogorov's energy spectrum, the large scale ones feel the effects of the pipe walls and the spectra are a little shallower. One can find that the energy dissipation rate, ϵ, is independent of viscosity. In the limit that $\nu \to 0$, the viscous length scale, L_ν, tends to zero, but ϵ itself does not. This is an important result. Just as the no slip condition holds for any non zero ν, the Navier-Stokes equations, even with a very tiny viscosity, behave vastly different from having ν identically equal to zero.

One should note that there are many other methods for obtaining Kolmogorov's energy spectrum, and the theory itself is not exact. Nonetheless, for high Reynold's number flow, the energy spectrum predicted by Kolmogorov is often observed to a high degree of accuracy. It is this type of spectrum that is a tell tale sign of turbulence in the interstellar medium.

3 The Interstellar Medium

The interstellar medium is typically a highly compressible, supersonic flow. The two types of flows, incompressible and compressible, give drastically different turbulent behavior, although as stated previously, the incompressible case still gives some useful results for the physics of the interstellar medium. An important number when talking about fluid flow is the dimensionless Mach number defined as:

$$\mathcal{M} = \frac{v}{c_s} \tag{9}$$

where v is the speed of the fluid and c_s is the speed of sound in the fluid. Supersonic flows (i.e. $\mathcal{M} > 1$) produce shocks, which are a type of propagating disturbance characterized by a sudden change in the characteristics of the medium (such as pressure, temperature, or speed) typically causing a compression of the fluid.

3.1 Diagnostics

Turbulence in the interstellar medium is a complex beast, but various diagnostics have been developed to identify relevant physical processes. The structure function, autocorrelation, power spectra, energy spectra, and delta variance are important tools for characterizing interstellar turbulence. A structure function for an observable A assuming isotropy is defined as:

$$S_p(\delta r) = \langle |A(r) - A(r + \delta r)|^p \rangle \tag{10}$$

for a position r and increment δr, with a power-law to fit this, $S_p(\delta r) \propto \delta r^{\zeta_p}$, gives the slope ζ_p. The brackets ($\langle \rangle$) denote a spatial average over the volume one is interested in. The structure function can be seen as sort of a measure of correlation. If the observable A is the velocity, it is like a measure of correlation of the scales of the velocities over the volume. If δr is very tiny one would expect the p^{th} moment to also be very small and so on. Another useful tool employing two-point statistics is the autocorrelation of A, defined as:

$$C(\delta r) = \langle A(r) A(r + \delta r) \rangle \tag{11}$$

Physically this function represents the spatial average of the product of the observable at a position r and a position $r + \delta r$. We can then deduce that $C(\delta r)$ says how much an observable at a different position depends on the observable at another position, hence the name given to this function. Another useful quantity is the power spectrum:

$$P(k) = \langle \hat{A}(k) \hat{A}(k)^* \rangle \tag{12}$$

where $\hat{A}(k)$ is the Fourier transform of $A(r)$ defined as $\hat{A} = \int e^{ikr} A(r) dr$ and A^* is the complex conjugate of A and now the brackets denote an average over k space. The power spectrum is related to the Kolmogorov energy spectrum discussed earlier by the relation

$$\mathcal{E}(k) = \int \hat{A}(k)\hat{A}(k)^* d^D k \tag{13}$$

where D is the spatial dimension. One finds that $\mathcal{E} \propto P(k)k^{D-1}$. For 1D, 2D, and 3D Kolmogorov energy spectra we have:

$$\begin{array}{ll} E \propto k^{-11/3} & \text{(1D)} \\ E \propto k^{-8/3} & \text{(2D)} \\ E \propto k^{-5/3} & \text{(3D)} \end{array}$$

The Delta variance is a technique to measure power and the relative amount of structure on various scales using a digital enhancement technique called unsharp mask:

$$\sigma_\Delta^2(L) = \langle \int_0^{3L/2} dx \left\{ (A[r+x] - \langle A \rangle) \bigodot(x) \right\}^2 \rangle \tag{14}$$

where

$$\bigodot(x) = \pi(L/2)^{-2} \begin{cases} 1 & 0 < x < L/2 \\ -1/8 & L/2 < x < 3L/2 \end{cases} \tag{15}$$

The delta variance is related to the power spectrum in that for an emission distribution with a power spectrum $\propto k^{-n}$, the delta variance is $\propto r^{n-2}$ for $r = 1/k$.

3.2 Observations

Maps of the interstellar medium observed in atomic and molecular line transitions, infrared and radio continuum emission, and dust extinction have shown a complex spatial distribution observed over scales greater than 50 pc to scales down to less than a parsec. Each step in higher spatial resolution has provided more details of the structure of the ISM. Velocity profiles can be observed on various scales from the Doppler shift of frequencies in the absoprtion profiles. These types of observations have supported the idea that the interstellar medium is inherently turbulent. Power spectra of 2D maps of Milky Way HI emission and absorption have been demonstrated to have power-law slopes of around -2.8 to -3.2. This is comparable to the result obtained for incompressible turbulence of -11/3. These observations have come from HI emission from the Large Magellanic Cloud (LMC) and dust spirals in galactic nuclei. The interstellar medium is apparently full of these power-law power spectra giving convincing evidence the that ISM is indeed a turbulent medium.

Turbulence decays in a time on the order of the dynamical time for the system. So why do we see turbulence everywhere in the ISM? The Kolmogorov energy spectrum was reached by considering some forcing source for the turbulence. Either the observable universe has not completed a dynamical time or there is some forcing source of turbulence in the ISM. What is the source of turbulence in the ISM? The physical processes by which kinetic energy gets converted into turbulence are actually not very well understood for the ISM. Some of the main sources of large scale motions are (a) stars, whose energy input to the ISM is in the form of protostellar winds, expanding HII regions, and supernovas (b) galactic rotation in the shocks of spiral arms (c) self-gravity through instabilities and cloud collapse (d) Kelvin-Helmholtz instability (the onset of instability and transition to turbulent flow in fluids of different densities moving at various speeds) and (e) galaxy interactions. Small scale turbulence observed by radio scintillation include sonic reflections and shock waves hitting clouds, cosmic ray streaming, field star motions and winds, and of course, energy cascades from larger scales.

Work has been done to quantify the energy input from large scale sources, but efforts have always required some sort of efficiency factor which cannot be explained from first principles. Many mechanisms have been proposed for the energy input to turbulence. For example, galactic rotation is seen as a virtually unlimited supply of energy for turbulence if it can be properly "tapped" into. Work has been done to try to understand how turbulence might tap into this energy, but nothing conclusive has resulted from this work as of yet. Sources of interstellar turbulence span such a large range of scales that it is often very difficult to even identify what it might be for a particular region. The details of the turbulence can be greatly affected by the type and scale of the energy input. There is still much to learn about turbulence that is driven like this.

4 Star Formation

Understanding star formation is of central importance to understanding the formation and evolution of galaxies. Much of modern astrophysics would greatly benefit from a quantitative prediction of the star formation rate and the stellar mass distribution. The process of star formation involves extremely complex physics characterized by the interaction of turbulence, self-gravity, and magnetic fields. It was traditionally thought that the star formation process was controlled by the interplay of gravity and magnetostatic support, along with neutral ion drift known as ambipolar diffusion. Recently, self gravity, magnetic fields, *and* turbulence have been seen as central to star formation, but observation and numerical work suggest that supersonic turbulent flows actually control star formation. In this section, the goal is to briefly give a history of star formation, discuss simulations and observations, and also discuss the idea of turbulence as a pressure.

4.1 Historical Overview of Star Formation

The traditional solution of the problem of star formation was gravitational fragmentation. Cores of molecular clouds greater than a certain critical density become gravitationally unstable, fragment, and begin to collapse. Sir James Jeans studied the growth of plane wave density perturbations in an infinite uniform medium with a finite pressure, but no rotation, magnetic fields, or turbulence, and showed that perturbations whose wavelength exceed a critical value are gravity dominated and grow exponentially. This length is called the Jean's length and can be expressed in an isothermal medium with uniform density ρ as

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}} \tag{16}$$

where G is the universal gravitational constant and $c_s = \sqrt{kT/m}$ is the speed of sound in the medium. A corresponding minimum mass can be estimated and this mass is called the Jean's mass:

$$M_J = \frac{\pi^{3/2} c_s^3}{G^{3/2} \rho^{1/2}} \tag{17}$$

This analysis comes from fairly simple arguments but can be misleading since it is mathematically inconsistent because it neglects the collapse of the background medium which may overwhelm the individual density fluctuation. The assumed equilibrium conditions do not solve the basic equations. This has been called the "Jean's swindle." Observations in the 1970's clearly showed deviations from the gravitational fragmentation model. The disk of the milky way contains $\sim 10^9 M_\odot$ of molecular gas, the region where star formation is known to occur. From the gravitational fragmentation model, one can obtain a free fall time of the gas absent other support:

$$t_{ff} = \frac{1}{\sqrt{G\rho}} \tag{18}$$

which is about 3×10^6 years for the Milky Way. This would give the Milky Way a blistering star formation rate of around 250 $M_\odot$ yr^{-1} which is much greater than the observed rate of about 3 $M_\odot$ yr^{-1}. The reason for this is one of the major unsolved mysteries of the interstellar medium. Additionally, supersonic turbulent motions appeared to occur in molecular clouds on all scales, which is quite different from the gravitational fragmentation model. Clearly, the fragmentation model is incomplete, although aspects of it still exist in most star formation models today.

Nonetheless, the complexity of the fragmentation process due to the effects of turbulence and magnetic fields has been known for some time. As stated previously, in the early 1950's, turbulence was known to be important in the ISM. Galactic magnetic fields were also known to exist due to observations of polarized starlight. Chandrasekhar studied the effects of turbulence and magnetic fields on gravitational instability. However, not until recently has a treatment of the fragmentation process including turbulence, magnetic fields, and self gravity been possible due to the nonlinear nature of the problem and inadequate computers. Today, we have both the computer resources and numerical techniques to directly simulate the fragmentation process on a large scale.

4.2 Observations and the Empirical Law

Since star formation is so prevalent in molecular clouds, it is important to know the mass of these clouds. The total molecular mass M_{TOT} is assumed to be proportional to the luminosity of a CO rotational transition ($M_{TOT} \propto L_{CO}$). The lowest order transition, $J = 1 \rightarrow 0$, has been used extensively for this purpose. For measuring the star formation rate $\dot{M}_*$, people have used the ultraviolet continuum emission, the integrated blue light, Hα emission, and far-infrared emission. The far infrared luminosity (L_{fir}) has been the most promising candidate for measuring the star formation rate. Various studies have concluded on the conversion factor from L_{fir} to $\dot{M}_*$. The conversion factors from these studies vary by a factor of three. As long as the $L_{CO} - M_{TOT}$ and $L_{fir} - \dot{M}_*$ relations are uncorrelated with each other, a plot of L_{fir} vs. L_{CO} should show how star formation rate varies with molecular mass. Also, the ratio L_{fir}/L_{CO} should be proportional to the star formation rate per unit mass of molecular gas.

These types of observations and analysis have led to the empirical Kennicutt-Schmidt Law, which can be stated in two forms, both consistent with observation:

$$\dot{\Sigma}_* \propto \Sigma_g^{1.4} \tag{19}$$

or

$$\dot{\Sigma}_* \propto \frac{\Sigma_g}{\tau_{dyn}} \tag{20}$$

where $\dot{\Sigma}_*$ is the star formation rate per unit area, Σ_g is the surface density of the gas, and τ_{dyn} is the dynamical time scale of the galactic disk. Schmidt proposed a relationship between gas density and star formation about 40 years before Kennicutt in 1998 determined the exponents and coeffiecients of the relations from a large galaxy sample. Both forms of the Kennicutt-Schmidt Law fit the observed samples of galaxies quite well over a range of eight orders of magnitude in star formation rate. In a recent paper from Krumholz and McKee, they claim to have obtained a general theory of turbulence regulated star formation in agreement with the Kennicutt-Schmidt Law and the surprisingly low actual star formation rate. In their paper, they discuss the interplay of gravity, thermal energy and turbulent pressure.

The concept of turbulent pressure has been difficult to avoid since turbulence first became important due to its convenience versus trying to solve the full blown equations. Turbulent pressure has been used to generalize gravitational instability, to confine molecular clouds, and to approximate an equation of state. However, the turbulent pressure approach has been shown to only be correct if the dominant scale is much smaller than the region under consideration and the Mach number is much less than unity. However, this is simply not the case. The turbulent energy spectrum places most of the energy on the largest scale (see the Kolmogorov $-5/3$ law), and, since the spectrum is continuous, the scale separation required for turbulent pressure to provide correct results does not exist. Additionally, supersonic turbulence is likely to be dominated by highly intermittent shocks whose effects are very difficult to model as a pressure.

4.3 Full Blown Turbulent Star Formation

One can see that using turbulence as a pressure is a dangerous business, thus limiting what one can do using analytical techniques. Numerical simulations of the hydrodynamical or magneto hydrodynamic (MHD) equations provide the only means to actually see interstellar turbulence in action. Padoan et al. solve the compressible MHD equations in a simulation using periodic boundary conditions to find that the mass distribution of collapsing cores is consistent with the steller initial mass function (IMF), a function that specifies the mass distribution of a newly formed stellar population. In these simulations (and others similar to them), the initial density and magnetic fields are uniform and the initial velocity is random, generated in Fourier space with power only on a large scale (small wave number). A random external force is applied to drive the turbulence at a roughly constant Mach number. Typically, self gravity is turned on only after the turbulence reaches a state of dynamical equilibrium. The main output of many numerical experiments are a large number of spatially resolved collapsing protostars which are identified using a peak-find algorithm. Once the protostars are identified, average values of the radial profiles of magnetic field strength, gas density, angular momentum, infall velocity and turbulent velocity dispersion are computed in order to validate the simulation results by comparison with observational data. Observable properties of the simulated protostars can be extracted in a way that mimics real observations using radiative transfer codes.

It is still not entirely clear whether the nature of the random external force driving the turbulence will have a major impact on the dynamics. The random external force is necessary however, because, as stated previously, turbulence will dissipate in a dynamical time without it. It was previously thought that magneto hydrodynamic waves provided the means to prevent the dissipation of interstellar turbulence, but numerical models have shown this is not the case.

Simulations today can be performed with sizes of 1024^3 and use advanced adaptive mesh refinement (AMR) techniques to zoom in on regions of interest. The James Webb Space Telescope, the successor to the Hubble Space Telescope, and the ground-based High Dynamic Range Telescope, among other huge telescope projects such as Spitzer, SOFIA, and Herschel will be able to peer at the first galaxies formed in the early universe. These observations will require a theoretical model of large-scale star formation to be fully understood. Such a model should be equally applicable to the early universe, protogalaxies, merging galaxies, and other environments as well as nearby interstellar clouds. Finding the mass distribution and star formation rate are at the forefront of current research in astrophysics. Numerical simulations employing fully developed turbulence are probably the most useful tool today to find a suitable theoretical model giving us great insight on the early universe.

5 Conclusion

Turbulence is one of the most fundamental problems in all of science today. A general solution of the Navier-Stokes equation has been elusive for over a hundred years and may very well be for hundreds more. Entirely new and deep mathematical ideas are likely needed to tackle this problem. Unlocking the secrets hidden in the Navier-Stokes equations is not a simple task, but studies of the interstellar medium and star formation have led to great insights into turbulence. The interstellar medium has been shown to be a very turbulent medium, and many of the physical processes may indeed be controlled by turbulence. Nonetheless, our understanding of turbulence is far from complete. Future studies of the ISM will likely lead to new insights important to understanding turbulence at a fundamental level.

6 References

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