Math

Vector Identities

$a \cdot (b \times c) = b \cdot (c \times a) = c \cdot (a \times b)$
$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$
$(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$
$\nabla \times \nabla\psi = 0$
$\nabla \cdot (\nabla \times a) = 0$
$\nabla \times (\nabla \times a) = \nabla(\nabla \cdot a) - \nabla^2 a$
$\nabla \cdot (\psi a) = a \cdot \nabla\psi + \psi\nabla \cdot a$
$\nabla \times (\psi a) = \nabla\psi \times a + \psi\nabla \times a$
$\nabla(a \cdot b) = (a \cdot \nabla)b + (b \cdot \nabla)a + a \times (\nabla \times b) + b \times (\nabla \times a)$
$\nabla \cdot (a \times b) = b \cdot (\nabla \times a) - a \cdot (\nabla \times b)$
$\nabla \times (a \times b) = a(\nabla \cdot b) - b(\nabla \cdot a) + (b \cdot \nabla)a - (a \cdot \nabla)b$

Vector Calculus Theorems

$\int_v \nabla \cdot A \; dV = \int_s A \cdot n \; dA$ (Divergence)
$\int_s (\nabla \times A) \cdot n \; dA = \oint_C A \cdot dl$ (Stokes)

Curvilinear Coordinates

$(x, y, x) \to h_1 = h_2 = h_3 = 1$
$(r, \phi, z) \to h_1 = 1 \; h_2 = r \; h_3 = 1$
$(r, \theta, \phi) \to h_1 = 1 \; h_2 = r \; h_3 = r\sin\theta$

$ds = \hat{e}_1 h_1 dx_1 + \hat{e}_2 h_2 dx_2 + \hat{e}_3 h_3 dx_3 \quad dV = h_1 dx_1 h_2 dx_2 h_3 dx_3$
$\nabla u = \hat{e}_1 \frac{1}{h_1}\frac{\partial u}{\partial x_1} + \hat{e}_2 \frac{1}{h_2}\frac{\partial u}{\partial x_2} + \hat{e}_3 \frac{1}{h_3}\frac{\partial u}{\partial x_3}$
$\nabla \cdot v = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial x_1}(h_2 h_3 v_1) + \frac{\partial}{\partial x_2}(h_1 h_3 v_2) + \frac{\partial}{\partial x_3}(h_1 h_2 v_3)\right]$

$$\nabla \times v = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} \\ h_1 v_1 & h_2 v_2 & h_3 v_3 \end{vmatrix}$$

$\nabla^2 u = \nabla \cdot \nabla u$
$x = r\cos\phi \qquad y = r\sin\phi \qquad z = z$
$\hat{r} = \cos\phi\hat{x} + \sin\phi\hat{y} \qquad \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y} \qquad \hat{z} = \hat{z}$
$\hat{x} = \cos\phi\hat{r} - \sin\phi\hat{\phi} \qquad \hat{y} = \sin\phi\hat{r} + \cos\phi\hat{\phi} \qquad \hat{z} = \hat{z}$
$x = r\sin\theta\cos\phi \qquad y = r\sin\theta\sin\phi \qquad z = r\cos\theta$
$\hat{r} = \sin\theta\cos\phi\hat{x} + \sin\theta\sin\phi\hat{y} + \cos\theta\hat{z} \qquad \hat{\theta} = \cos\theta\cos\phi\hat{x} + \cos\theta\sin\phi\hat{y} - \sin\theta\hat{z} \qquad \hat{\phi} = -\sin\phi\hat{x} + \cos\phi\hat{y}$
$\hat{x} = \sin\theta\cos\phi\hat{r} + \cos\theta\cos\phi\hat{\theta} - \sin\phi\hat{\phi} \qquad \hat{y} = \sin\theta\sin\phi\hat{r} + \cos\theta\sin\phi\hat{\theta} + \cos\phi\hat{\phi} \qquad \hat{z} = \cos\theta\hat{r} - \sin\theta\hat{\theta}$

Series

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$
$(a+b)^n = \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} a^{n-k} b^k$
$f(a+x) = f(a) + xf'(a) + \frac{x^2}{2!}f''(a) + \ldots$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$
$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$
$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$
$\sum_{n=0}^{N} x^n = \frac{1-x^{N+1}}{1-x}$

Complex Analysis

$z = x + iy = re^{i\theta} = r(\cos\theta + i\sin\theta) \quad f(z) = u(x,y) + iv(x,y)$
Cauchy-Riemann: $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

Residue Calculus

The following methods may be used to calculate the residue of a function F(z) at a singular point z_0.
1. Expand F(z) in a series about z_0, and so obtain the coefficient of the term $1/(z - z_0)$. This fundamental method uses the definition of the residue and is valid for all types of singularities.
2. If the point z_0 is a pole of order m, the residue may be calculated by taking the following limit:
$R = lim_{z\to z_0} \frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}[(z - z_0)^m F(z)]$
3. If the point z_0 is a simple pole, the residue may be calculated by taking the following limit:
$R = lim_{z\to z_0}(z - z_0)F(z)$
4. If F(z) may be put in the form $F(z) = p(z)/q(z)$ where $q(z_0) = 0$ but $\left.\frac{dq}{dz}\right|_{z=z_0} \neq 0$. and where $p(z_0) \neq 0$, the residue may be calculated by taking the following limit:
$R = lim_{z\to z_0} \frac{p}{dq/dz}$

$\oint_C f(z)dz = 2\pi i \left(\sum residues\right)$

Series Summation

$\sum_{n=-\infty}^{\infty} (-1)^n f(n) \to \oint_C \frac{f(z)dz}{\sin \pi z}$
$\sum_{n=-\infty}^{\infty} f(n) \to \oint_C \frac{\pi}{\tan \pi z} f(z)$
$\sum_{n=-\infty}^{\infty} f(in) \to \oint_C \frac{f(z)}{e^z - 1}$

Gamma Function

$\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt \quad \Re(z) > 0$
$\Gamma(z+1) = z\Gamma(z) \qquad \Gamma(z)\Gamma(1-z) = \frac{\pi}{\sin \pi z}$
$\Gamma(n) = (n-1)!$

Error Propogation

$q = x + \ldots + z - (u + \ldots + w)$:
$\delta q = \sqrt{(\delta x)^2 + \ldots + (\delta z)^2 + (\delta u)^2 + \ldots + (\delta w)^2}$
$q = \frac{x \times \ldots \times z}{u \times \ldots \times w}$:
$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta x}{x}\right)^2 + \ldots + \left(\frac{\delta z}{z}\right)^2 + \left(\frac{\delta u}{u}\right)^2 + \ldots + \left(\frac{\delta w}{w}\right)^2}$
$q = Bx$:
$\delta q = |B|\delta x$
$q = x^n$:
$\frac{\delta q}{|q|} = |n|\frac{\delta x}{|x|}$
$q = q(x, \ldots, z)$:
$\delta q = \sqrt{\left(\frac{\partial q}{\partial x}\delta x\right)^2 + \ldots + \left(\frac{\partial q}{\partial z}\delta z\right)^2}$

Special Functions

Bessel: $e^{(x/2)(t-1/t)} = \sum_{n=-\infty}^{\infty} J_n(x)t^n$
Legendre: $(1 - 2xt + t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$
Hermite: $e^{-t^2+2tx} = \sum_{n=0}^{\infty} H_n(x)\frac{t^n}{n!}$
Laguerre: $\frac{e^{-xz/(1-z)}}{1-z} = \sum_{n=0}^{\infty} L_n(x)z^n$

Asymptotic Expansion of Integrals

$I(x) = \int_a^b f(t)e^{x\psi(t)}dt \qquad x \to \infty$
if ψ has a max at $c, \psi'(c) = 0 \qquad a < c < b$
then $\psi(t) = \psi(c) + \underbrace{\psi'(c)(t-c)}_{=0} + \frac{\psi''(c)}{2!}(t-c)^2 + \ldots$

$I(x) \approx \int_a^b f(c)e^{x\psi(c)}e^{x\frac{\psi''(c)}{2!}(t-c)^2}dt \qquad \psi''(c) < 0$ for max
$= \int_{c-\epsilon}^{c+\epsilon} f(c)e^{x\psi(c)}e^{-x\frac{|\psi''(c)|}{2!}(t-c)^2}$
$= f(c)e^{x\psi(c)}\int_{-\infty}^{\infty} e^{-x\frac{|\psi''(c)|}{2!}(t-c)^2}dt = \frac{\sqrt{2\pi}f(c)e^{x\psi(c)}}{\sqrt{x|\psi''(c)|}}$

if the max occurs at the one of the endpoints, it is half of the calculated value
if $\psi(t)$ has no maximum in the interval (a, b) then take the max value to be at endpoint which maximizes $\psi(t)$, notice here that $\psi'(c) \neq 0$ so the second derivative is not needed

Other Useful Things

$\int_0^\infty e^{-ax^2} dx = \frac{1}{2} \left(\frac{\pi}{a} \right)^{1/2}$
$\ln n! \simeq n \ln n - n$

Fourier Transforms

Trig Identities